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Peristaltic Transport of a Herschel-Bulkley Fluid in Contact With a Newtonian Fluid in a Circular Tube

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Abstract

Peristaltic flow of a Herschel-Bulkley fluid in contact with a Newtonian fluid in a circular tube is studied. The interface which is a stream line is determined by solving a non-linear equation. The expressions for the velocity, the stream function and the pressure rise are obtained. The effects of different parameters on the flow phenomenon are discussed using graphical representation. Here some interesting results were obtained which warrant further investigation on the flow of two immiscible fluids in a circular tube with flexible walls. High amplitude gives rise to a thicker core layer in the first half wave length of the tube region and high amplitude gives rise to a thinner core layer in the second half wave length region of the tube.

Keywords: Peristalsis, Stream function, Interface.

Introduction

Physiological fluids in human and animal bodies are pumped by the continuous periodic muscular oscillations of the ducts. These oscillations are presumed to be caused by the progressive transverse contraction waves that propagate along the walls of the ducts. Peristalsis is the mechanism of the fluid transport that occurs generally from a region of lower pressure to higher pressure when a progressive wave of area contraction and expansion travels along the flexible wall of the tube. Peristaltic flow occurs widely in the functioning of the ureter, food mixing and chyme movement in the intestine. There are many important applications of this principle such as the design of roller pumps, which are useful in pumping fluids without contamination due to contact with the pumping machinery.

In 1966, Latham made the first experimental study on the mechanics of peristaltic transport. The results of the experiments were found to be in good agreement with the theoretical results of Shapiro[1]. Based on this experimental work, Burns and Parkes [2] studied the peristaltic motion of a viscous fluid through a pipe and a channel by considering sinusoidal variations at the walls. Shapiro et al. [3] analyzed the peristaltic pumping with long wave length and low Reynolds number assumptions. The small Reynolds number assumption of Shapiro et al. [4] was endorsed by Jaffrin [5], who extended the analysis by considering the higher order terms to include cases where the Reynolds number was higher. Barton and Raynor [6] studied peristaltic flow in tubes using long wavelength approximations. Peristaltic waves in circular cylindrical tubes were analyzed by Yin and Fung [7] and a complete review of peristaltic transport is given by Jaffrin and Shapiro [8]

We know, that a Herschel-Bulkley fluid is a semisolid rather than an actual fluid. We know by definition such a fluid cannot support a shear stress. This implies that materials that have an yield stress are not fluids such materials are referred to as Yield stress fluids. A detailed discussion of the inappropriateness of the use of such models for fluids is discussed in the recent review paper by Krishnan and Rajagopal [9]. While such materials might not to be fluids, there is value in studying them as they give some idea of the behavior of fluids of interest under certain limits.

Radha Krishnama Charya [10] discussed Perisistaltic motion of a power - low fluid using long wave length approximations. Also Radha Krishnama Charya et al. [11] analysed heat transfer the peristaltic transport in a non-uniform channel. Srinivas et al. [12] studied the influence of heat and mass transfer on MHD peristaltic flow through a porous space with complaint walls. Mixed convective heat and mass transfer in an asymmetric channel with peristalsis is investigated by Srinivas et al. [13].

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Among models of semisolids the Herschel-Bulkley model is preferable because it describes blood behavior very closely and Newtonian, Bingham and Power-law models can be derived as special cases. Vajravelu et al. [14] studied the peristaltic motion of Herschel-Bulkley fluid in contact with a Newtonian fluid between flexible rigid walls. Recently Vajravelu et al. [15] investigated the peristaltic flow of Casson fluid in contact with a Newtonian fluid between permeable layers. Narahari et al. [16] studied the peristaltic transport of Bingham fluid in contact with a Newtonian fluid.

Motivated by these studies axisymmetric analysis is made for the peristaltic pumping of a two fluid system (Herschel-Bulkley fluid in contact with a Newtonian fluid) in a circular tube with flexible wall. The interface which is a stream line is determined by solving a non-linear equation. The expressions for the velocity, the stream function and the pressure rise are obtained. Some deductions are made and they are found to agree with earlier works.

Nomenclature

τ_0	Yield stress
u_1	Velocity of the core layer
u_2	Velocity of the peripheral layer
μ_1	Viscosity of the Herschel-Bulkley fluid
μ_2	Viscosity of the Newtonian fluid
p	Pressure
u_p	Plug flow velocity
r_p	Plug radius
U	Average radius of the tube
h_1	Interface

Mathematical Formulation of the Problem

Consider the peristaltic transport of a Herschel-Bulkley in contact with a Newtonian fluid in a tube of radius 'a'. The core region of the tube contains Herschel-Bulkley fluid where as the peripheral region is occupied by a Newtonian fluid. The flow is axisymmetric. The axisymmetric geometry facilitates the choice of cylindrical polar coordinate system (R, Θ, Z) to study the problem. The wall deformation due to the propagation of an infinite train of peristaltic waves is represented by

$$R = H(Z,t) = a + bSin\frac{2\pi}{\lambda}(Z - ct) \qquad (1)$$

where b is the amplitude, λ is the wavelength and c is the wave speed.
Under the assumptions that tube length is an integral multiple of the wavelength λ and the pressure across the ends of the tube is a constant, the flow is inherently unsteady in the laboratory frame (R, θ, z) which is moving with velocity c along the wave. The transformation between these two frames is given by

$$r = R, \quad \theta = \Theta \quad, z = Z - ct, \; p(z) = p(Z,t) \qquad (2)$$

The basic equations governing the motion of the fluids are:

$$\frac{\partial u_p}{\partial r} = 0, \qquad 0 \le r \le r_p \qquad (3)$$

$$\frac{\partial p}{\partial z} = \frac{-1}{r}\frac{\partial}{\partial r}\left\{r\left[\mu_1\left(-\frac{\partial u_1}{\partial r}\right)^n + \tau_0\right]\right\}, \quad r_p \le r \le h_1 \qquad (4)$$

$$\frac{\partial p}{\partial z} = \frac{-1}{r}\frac{\partial}{\partial r}\left\{r\left[-\mu_2\left(\frac{\partial u_2}{\partial r}\right)\right]\right\}, \quad h_1 \le r \le h \qquad (5)$$

where τ_0 is the yield stress.
and the corresponding boundary conditions are

$u_1 = u_p$ at $r = r_p$
$u_1 = u_2$ at $r = h_1$
$\tau_1 = \tau_2$ at $r = h_1$
$u_2 = -c$ at $r = h$

Nondimensionalization of the Flow Quantities

The following non-dimensionalized quantities are introduced to make the basic equations and the boundary conditions dimensionless:

$$\bar{u}_i = \frac{u_i}{c}, \bar{z} = \frac{z}{\lambda}, \bar{r} = \frac{r}{a}, \bar{h} = \frac{h}{a}, \bar{h}_i = \frac{h_i}{a}, \bar{t} = \frac{c}{\lambda}t, \bar{p} = \frac{a^{n+1}}{c^n \lambda \mu_2} p,$$

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$\bar{\tau}_0 = \frac{a^n}{c^n \mu_2}\, \tau_0, \bar{\tau}_2 = \frac{a^n}{c^n \mu_2}\, \tau_2, \bar{\tau}_3 = \frac{a^n}{c^n \mu_2}\, \tau_3\ , \mu = \frac{\mu_3}{\mu_2},\ \phi = \frac{b}{a}$ (6)

The equations governing the motion become (ignoring the bars)

$$\frac{\partial u_p}{\partial r} = 0 \quad (7)$$

$$\frac{\partial p}{\partial z} = \frac{-1}{r}\frac{\partial}{\partial r}\left\{ r\left[\left(-\frac{\partial u_1}{\partial r} \right)^n + \tau_0 \right] \right\} \quad (8)$$

$$\frac{\partial p}{\partial z} = \frac{-1}{r}\frac{\partial}{\partial r}\left\{ r\left[-\mu\left(\frac{\partial u_2}{\partial r} \right) \right] \right\} \quad (9)$$

The boundary conditions in dimensionless form are

$u_1 = u_p$ at $r = r_p$ (10)

$u_1 = u_2$ at $r = h_1$ (11)

$\tau_1 = \tau_2$ at $r = h_1$ (12)

$u_2 = -1$ at $r = h$ (13)

Solution of the Problem

Solving the equations (7)-(9) subject to the boundary conditions (11) - (13) we get,

$$u_1 = \frac{P}{4\mu}(h^2 - {h_1}^2) - 1 + \frac{2n}{n+1}\frac{1}{P}\left[\left(\frac{Ph_1}{2} - \tau_0 \right)^{\frac{1}{n}+1} - \left(\frac{Pr}{2} - \tau_0 \right)^{\frac{1}{n}+1} \right],\ r_p \le r \le h_1 \quad (14)$$

$$u_2 = \left(\frac{h^2 - r^2}{4\mu} \right) P - 1, \qquad h_1 \le r \le h \quad (15)$$

and the plug velocity is obtained as

$$u_p = \frac{1}{4\mu}(h^2 - {h_1}^2)P - 1 + \frac{2n}{n+1}\frac{1}{P}\left[\left(\frac{Ph_1}{2} - \tau_0 \right)^{\frac{1}{n}+1} - \left(\frac{Pr_p}{2} - \tau_0 \right)^{\frac{1}{n}+1} \right] 0 \le r \le r_p \quad (16)$$

where $P = -\frac{\partial p}{\partial z}$

The flow rate q across any cross section is independent of z under lubrication approach and is given by

$$q = q_p + q_1 + q_2 = 2\int_0^{r_p} r u_p dr + 2\int_{r_p}^{h_1} r u_1 dr + 2\int_{h_1}^{h} r u_2 dr$$

$$q = \frac{{r_p}^2}{4\mu}(h^2 - {h_1}^2) - h^2 + \frac{4}{P(k+1)}\left(\frac{Ph_1}{2} - \tau_0 \right)^{k+1}\left[\frac{{h_1}^2}{2} - \frac{2h_1}{P(k+1)}\left(\frac{Ph_1}{2} - \tau_0 \right) + \frac{4}{P^2(k+2)(k+3)}\left(\frac{Ph_1}{2} - \tau_0 \right)^2 \right] + \frac{P}{4\mu}\left[2h^2{h_1}^2 + {r_p}^2{h_1}^2 - {r_p}^2h^2 - \frac{3{h_1}^4}{2} - \frac{{h_1}^4}{2} \right] \quad (17)$$

where q_1 and q_2 are the core and peripheral layer flow rates respectively. The dimensionless average volume flow rate $\overline{Q}$ over one wavelength is obtained as

$$\bar{Q} = 2\int_0^1 \int_0^h r(u_i + 1)drdt = q + \int_0^1 h^2 dt \quad (18)$$

where $u = u_1\ for\ 0 \le r \le h_1\ , u = u_2\ for\ h_1 \le r \le h$

We define $u_i = \frac{1}{r}\frac{\partial \psi_i}{\partial r}\ and\ v_i = -\frac{1}{r}\frac{\partial \psi_i}{\partial z}$ u_i and v_i are axial and radial velocities respectively.

The solutions in terms of the stream functions can be obtained by using the conditions $\psi_p = 0\ at\ r = 0\ , \psi_2 = q/2\ at\ r = h\ , \psi_2 = \psi_p\ at\ r = r_p$ in (14) and (15) They are given by

$$\psi_p = \frac{r^2}{2}\left[-1 + \frac{p}{4\mu}(h^2 - h_1^2) + \frac{2n}{(n+1)p}\left(\frac{ph_1}{2} - \tau o \right)^{\frac{1}{n}+1} - \frac{2n}{(n+1)p}\left(\frac{Ph_p}{2} - \tau o \right)^{\frac{1}{n}+1} \right], 0 \le r \le r_p$$

$$\psi_1 = B_1 - \frac{r^2}{2} + \frac{p}{4\mu}\left(h^2 - h_1^2\right)\frac{r^2}{2} + \frac{P^K}{2^K(K+1)}\left\{\frac{r^2}{2}.\left(h_1 - r_p\right)^{K+1} - \frac{r\left(r-r_p\right)^{K+2}}{K+2} + \frac{\left(r-r_p\right)^{K+3}}{(K+2)(K+3)}\right\}$$

where $B_1 = \frac{r.r_p^2}{(n+1)P}\left[\left(\frac{Ph_1}{2} - \tau o\right)^{k+1} - \left(\frac{pr_p}{2} - \tau o\right)^{k+1}\right] - \frac{p^K}{2^K(K+1)}\frac{r_p^2}{2},\left(h_1 - r_p\right)^{K+1}$

$$\psi_2 = \frac{q}{2} + \left(\frac{h^2 - r^2}{2}\right) + \frac{P}{4\mu}\left(\frac{h^2 r^2}{2} - \frac{r^4}{4} - \frac{h^4}{4}\right)$$

where $P = -\frac{\partial p}{\partial z}$ *and* $K = \frac{1}{n}$ (19*a*&*b*)

The interface

The equation for the interface is obtained from the condition $\psi_1 = \frac{q_1}{2}$ at $r = h_1$. Substituting in equation (19a) we have,

$$Q_1 = 2B_1 + \frac{P}{4\mu}h_1^2\left(h^2 - h_1^2\right) + \frac{P^K}{2^{K-1}(K+1)}\left\{\frac{h_1^2}{2}\left(h_1 - r_p\right)^{K+1} - \frac{h_1\left(h_1 - r_p\right)^{K+1}}{K+2} + \frac{\left(h_1 - r_p\right)^{K+2}}{(K+2)(K+3)}\right\} \quad (20)$$

Using the condition $\psi_2 = \frac{q_1}{2}$ at $r = h_1$ in equation (19b), the pressure gradient is obtained as

$$\frac{\partial p}{\partial z} = \frac{8\mu(Q - Q_1)}{2{h_1}^2 h^2 - {h_1}^4 - h^4} \quad (21)$$

Integrating the above equation with respect to z we obtain the pressure as:

$$\Delta p = 8\mu Q I_1 - 8\mu Q_1 I_1 \; 8\mu(Q - Q_1)I, \quad (22)$$

$$where \;\; I_1 = \int_0^1 \frac{dz}{2{h_1}^2 h^2 - {h_1}^4 - h^4}$$

The equation for the interface is given by

$$2B_1 - \frac{2\left(Q - Q_1\right)h_1^2\left(h^2 - h_1^2\right)}{\left(2h_1^2 h^2 - h_1^4 - h^4\right)} + \left\{\frac{-8\mu\left(Q - Q_1\right)}{\left(2h_1^2 h - h_1^4 - h^4\right)}\right\}^k \frac{1}{2^{K+1}(K+1)}\left\{\frac{h_1^2\left(h_1 r_p\right)^{K+1}}{2} - \frac{h_1\left(h_1 - r_p\right)^{K+1}}{K+2} + \frac{\left(h_1 - r_p\right)^{K+3}}{(K+2)(K+3)}\right\} = Q_1$$

(23)

where $Q_1 = q_1 + {h_1}^2$, $k = \frac{1}{n}$ and $Q = q + h^2$

The equation (23) has to he solved for h_1 for each z in the interval (0, h(z)) where h(z)=1+ϕ sin 2πz, the core flow rate $q_1\left(or\, Q_1\right)$ is determined using the condition $h_1 = \alpha \; at \; z = 0$ in the above equation.

This expression $q_1\left(or\, Q_1\right)$ is used in the equation (23) and the interface $h_1(z)$ is calculated numerically using Mathematica package.

Results and Discussions

The peristaltic transport of Herschel-Bulkley fluid in contact with a Newtonian fluid in a circular tube is investigated. The effects of different parameters on the velocity, the interface and the pressure rise are discussed. The equation of the interface is also obtained.

From Fig 2 it is observed that as the ratio of viscosity increases, the velocity of the fluid flow is decreasing. Fig 3 shows the variation of velocity with radius for different values of amplitude ratio. Here we notice that as the amplitude ratio increases, the velocity is increasing, further the effect of the amplitude ratio is more in the peripheral region when compared with the core region. Fig (4) infer that as the Power-law index is increasing the velocity is decreasing in the region $0 \le r \le 0.5$ and there no significant change in the region $0.6 \le r \le 1.5$.

The shape of the interface for different values of ratio of viscosity is shown in Fig 5. Higher values of ratio of viscocities gives rise to a thicker core layer in the first half wave length of the tube region and higher values of ratio of viscocities gives rise to a thicker peripheral layer in the second half wave length region of the tube. The shape of the interface for different values of power-law index is shown in Fig 6. Here we observe that there is no significant effect of power-law index in the first half wave length of the tube region and low value of power-law index, n, gives rise to a thinner peripheral layer in the second half wave length region of the tube. The shape of the interface for different amplitude ratios is shown in Fig 7. High amplitude gives rise to a thicker core layer in the first half wave length of the tube region and high amplitude gives rise to a thinner core layer in the second half wave length region of the tube. The shape of the interface for different values of plug radius is shown in Fig 8. Here we observe that the effect of plug radius has no significant change in the first half wave length of the tube region and high plug radius gives rise to a thicker core layer in the second half wave length region of the tube. Fig 9 shows the variation of pressure rise with flux for different values of amplitude ratio. Here we observe that as the amplitude ratio increases, the pressure is increasing and also we observe that as the flux increases the pressure is decreasing.

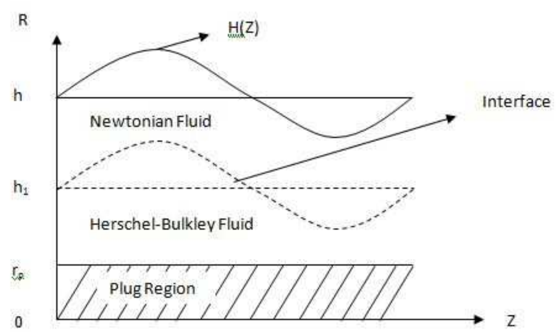

Fig 1: Physical Model

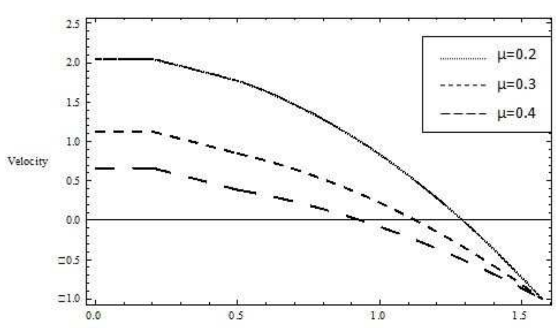

Fig 2: Variation of velocity with radius for different values of ratio of viscosities with the values of P=-1, τ=0.5, k=0.2, h1= 0.5, z=0.3, φ=0.6 and r_p=0.2.

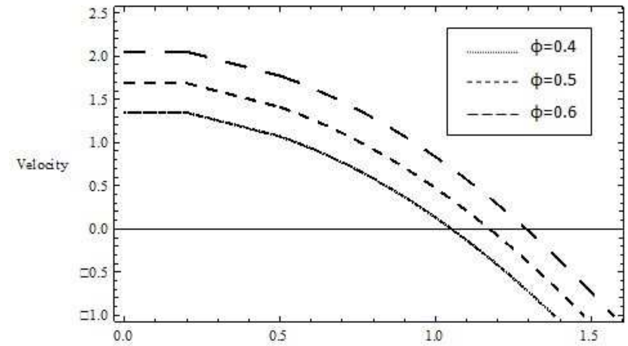

Fig 3: Variation of velocity with radius for different values of amplitude ratio with the values of P=-1, τ=0.5, k=0.2, h1= 0.5, z=0.3, μ=0.2 and r_p=0.2.

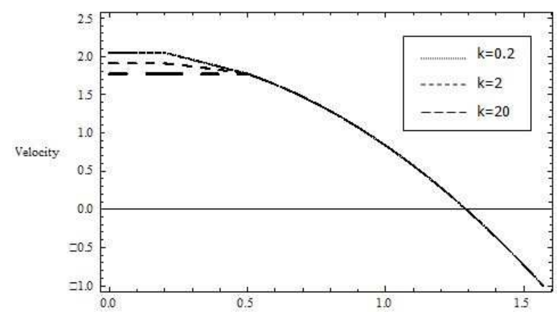

Fig 4: Variation of velocity with radius for different values of Power-law index with the values of P=-1, τ=0.5, φ=0.6, h1= 0.5, z=0.3, μ=0.2 and r_p=0.2.

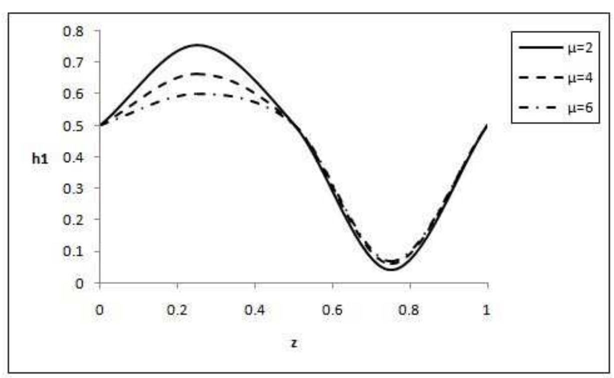

Fig 5: The shape of the interface for different values of ratio of viscosity with the values $\phi = 0.4; P = -1, k = 2, r_p = 0.3, Q = 0.1$

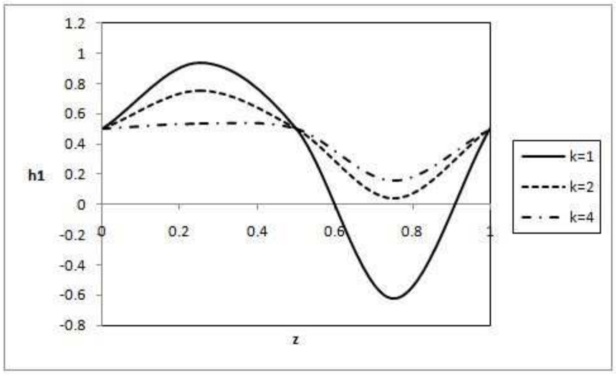

Fig 6: The shape of the interface for different values of ratio of power-law index with the values $\phi = 0.4;, P = -1, \mu = 2, r_p = 0.3, Q = 0.1$

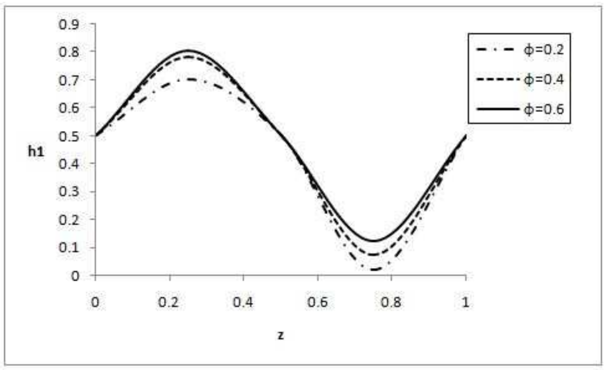

Fig 7: The shape of the interface for different values of amplitude ratio with the values $\phi = 0.4; P = -1, \mu = 2, r_p = 0.3, Q = 0.1$

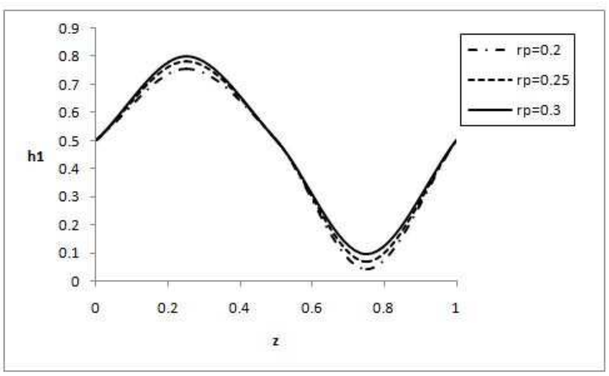

Fig 8: The shape of the interface for different values of plug radius with the values $\phi = 0.4; P = -1, \mu = 2, k = 2, Q = 0.1$

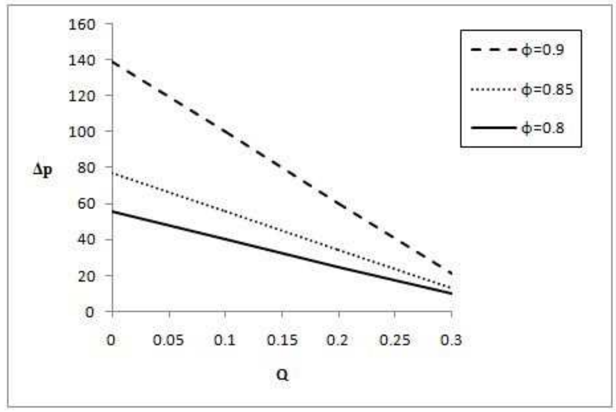

Fig 9: Variation of Δp with Q for different values of amplitude ratio for fixed values of k=2, μ=2, τ=0.2.

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